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Propagation of ultrasound near the phase transition in Ising systems

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Abstract. The effects of order-parameter strain coupling on the critical (tricritical) sound attenuation and dispersion are studied above T_c in the elastically isotropic Ising model. Under free boundary conditions a cross-over in the behaviour of acoustic quantities to a 'strong-coupling' regime is predicted. New scaling relations for the critical attenuation and dispersion are obtained and the corresponding scaling functions are calculated using an ε-expansion. Non-asymptotic effects are also discussed.

1. Introduction

Anomalies in sound attenuation and dispersion give us a valuable insight into the dynamics of the order-parameter (OP) fluctuations near the phase transitions in solids. These phenomena have been studied by a number of authors in the random-phase approximation (Pytte 1970), using scaling arguments and the mode-coupling theory (Schwabl 1973, Kawasaki 1976) and renormalisation group (RNG) theory (Murata 1976, Bhattacharjee 1982, Iro and Schwabl 1983, Dengler and Schwabl 1987). It was found that the critical exponents related to sound attenuation depend on the symmetry of the coupling between the strain and OP (Murata 1976, Fossum 1985). Recently, much attention has been paid to the problem of dynamic scaling of sound (Fossum 1985, Iro and Schwabl 1983, Dengler and Schwabl 1987). Iro and Schwabl (1983) have calculated the dynamic scaling functions for sound attenuation and dispersion above the critical temperature T_c. They have employed a generalised Nelson method of integration for the RNG recursion relations (Nelson 1976). Both the critical exponents and the scaling functions have been obtained by evaluating the OP fluctuations in a rigid lattice. In these 'weak-coupling' theories the sound mode is treated only as a small perturbation acting on the OP system.

The aim of this paper is to show that the inclusion of back-reaction of elastic degrees of freedom onto the OP considerably changes the critical exponents and the corresponding scaling functions. We shall limit our discussion to the disordered phase and to the case of scalar order parameter coupled with an isotropic elastic medium. In this case the elastic deformations are only coupled with the energy density of the OP and therefore the specific heat exponent α will play the crucial role in our considerations (Sak 1974).

The problem of the influence of elastic degrees of freedom on the statics of similar systems has been discussed in detail by Bergman and Halperin (1976) and by Bruno and

Sak (1980) using the RNG methods. They found that in isotropic systems with free boundaries (constant pressure) a first-order transition can be expected whenever the specific heat of the incompressible model diverges, as is the case for the Ising model in three dimensions. But since this discontinuity is usually small then the change in the critical behaviour can be observed probably only extremely close to T_c and one can observe the pseudo-critical or pseudo-tricritical behaviour in a range of temperatures prior to the transition, with ideal Ising exponents for the OP singularities. The first-order transition is associated with macroscopic instability, which appears when the bulk modulus becomes negative (Bergman and Halperin 1976). It is known that the critical behaviour of such a system depends strongly on the boundary conditions, and thus the Fisher-renormalised critical exponents are expected under pinned boundary conditions. The critical behaviour of the elastically isotropic Ising model is unstable with respect to anisotropic perturbations, regardless of boundary conditions. However, if the elastic anisotropy is small, one may expect that at a fixed pressure the system will encounter the macroscopic instability before the microscopic instability will develop. Then the system will behave in the same way as an exactly isotropic one.

So far, the effect of elastic couplings on sound damping has been studied in uniaxial dipolar systems by Meissner and Pirc (1980). Employing periodic boundary conditions (constant volume) they have predicted a logarithmic cross-over of the divergence of sound damping coefficient from a high-temperature 'rigid' regime to a 'compressible' regime.

In this paper we present the calculations of sound attenuation and dispersion in a compressible Ising-like system under a constant pressure. We find two different asymptotic pseudo-critical (pseudo-tricritical) regimes for acoustic properties: a weak-coupling ('rigid') one to which the lowest order in the elastic coupling theory applies and a strong-coupling ('compressible') regime with a large value of effective compressibility and with the ratio of shear and longitudinal bare moduli tending to zero. In the latter region the exponent governing the sound attenuation is smaller by $\alpha/2$ than that for the former case. We also describe a cross-over between these regimes and macroscopic instability effects in terms of two non-universal parameters.

The paper is organised as follows. In § 2 the model is presented and a general expression for the acoustic response function is derived. The following section (§ 3) is devoted to the discussion of the asymptotic results. In § 4 the cross-over and effects of macroscopic instability are studied. The application of the theory to the tricritical behaviour is discussed briefly in § 5. The last section (§ 6) gives the summary of the results.

2. The model and the acoustic response function

We consider an isotropic Ising model described by the Hamiltonian (Bruce 1980)

$$H = H_{\mathrm{el}} + H_{\mathrm{op}} + H_{\mathrm{int}} \tag{2.1}$$

consisting of three parts. The elastic part is

$$H_{\mathrm{el}} = \tfrac{1}{2}\int \mathrm{d}^d x\Big(C_{11}^0 \sum_{\alpha} e_{\alpha\alpha}^2 + 2C_{12}^0 \sum_{\alpha<\beta} e_{\alpha\alpha}e_{\beta\beta} + 4C_{44}^0 \sum_{\alpha<\beta} e_{\alpha\beta}^2\Big) \tag{2.2}$$

where the components of the strain tensor $e_{\alpha\beta}(\boldsymbol{x})$ are related to the displacement vector $\boldsymbol{u}(\boldsymbol{x})$ via

$$e_{\alpha\beta}(\boldsymbol{x}) = \tfrac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right) \tag{2.3}$$

and $C^0_{\alpha\beta}$ are the bare elastic moduli. For an isotropic medium $C^0_{44} = \tfrac{1}{2}(C^0_{11} - C^0_{12})$. For the OP part we shall adopt the standard Ginzburg–Landau form

$$H_{\mathrm{op}} = \int \mathrm{d}^d x[\tfrac{1}{2}r_0 S^2 + \tfrac{1}{2}(\nabla S)^2 + \tfrac{1}{4}\tilde{u}_0 S^4] \tag{2.4}$$

with the OP denoted by $S(\boldsymbol{x})$. Finally the term

$$H_{\mathrm{int}} = \int \mathrm{d}^d x\, g_0 \sum_\alpha e_{\alpha\alpha} S^2 \tag{2.5}$$

describes the interaction between the two fields.

A given elastic configuration can be separated into a homogeneous deformation and the constant-volume phonon parts (Larkin and Pikin 1969):

$$\frac{\partial u_\alpha}{\partial x_\beta} = u^0_{\alpha\beta} + (V\bar{\rho})^{-1/2} \sum_{k\neq 0} \mathrm{i}k_\beta u_\alpha(\boldsymbol{k}) \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}) \tag{2.6}$$

where V and $\bar{\rho}$ are the volume and density of the system at equilibrium. Then the normal-mode expansion is introduced:

$$\boldsymbol{u}(\boldsymbol{k}) = \sum_\lambda \boldsymbol{e}_\lambda(\boldsymbol{k}) Q_{k,\lambda} \tag{2.7}$$

with the normal coordinate $Q_{k,\lambda}$ and polarisation vector $\boldsymbol{e}_\lambda(\boldsymbol{k})$.

We would like to consider a system under constant external pressure P; therefore the term $V\Sigma_\alpha e_{\alpha\alpha}(P - P_0)$, where P_0 characterises the equilibrium state against which the strain is determined, should be added to the Hamiltonian.

At this stage one can integrate over the homogeneous deformations, which is equivalent to choosing free boundary conditions. The result is a shift of the mean-field transition temperature and a new term

$$-\frac{w_0}{4V}\left(\int \mathrm{d}^d x\, S^2\right)\left(\int \mathrm{d}^d y\, S^2\right)$$

added to the Hamiltonian, with $w_0 = 2g_0^2/B_0$ and

$$B_0 = C^0_{11} - \frac{2(d-1)}{d}(C^0_{11} - C^0_{12}).$$

The effective Hamiltonian will be denoted as $\bar{H}$.

We assume that the dynamics of the system is described by the Langevin equations (Meissner 1980, Schwabl and Iro 1981)

$$\dot{S}_k = -\Gamma_0 \frac{\delta\bar{H}}{\delta S_{-k}} + \xi_k \tag{2.8a}$$

$$\ddot{Q}_{k,\lambda} = -\frac{\delta\bar{H}}{\delta Q_{-k,\lambda}} - D^0_\lambda k^2 \dot{Q}_{k,\lambda} + \eta_{k,\lambda} \tag{2.8b}$$

where the Gaussian white noises $\xi_k(t)$ and $\eta_{k,\lambda}(t)$ have variances related to the bare damping terms Γ_0 and $D_\lambda^0 k^2$ by the Einstein relations.

It is convenient to represent these equations in a functional form (Martin *et al* 1973, Janssen 1976) with Lagrangian given by

$$L[S, \tilde{S}, Q, \tilde{Q}] = \sum_k \int \mathrm{d}t \Bigg\{ \sum_\lambda \Bigg[\tilde{Q}_{k,\lambda} D_\lambda^0 k^2 \tilde{Q}_{-k,\lambda} - \tilde{Q}_{k,\lambda} \times \left(\ddot{Q}_{-k,\lambda} + \frac{\delta \bar{H}}{\delta Q_{k,\lambda}} + D_\lambda^0 k^2 \dot{Q}_{-k,\lambda} \right) \Bigg] + \tilde{S}_k \Gamma_0 \tilde{S}_{-k} - \tilde{S}_k \times \left(\dot{S}_{-k} + \Gamma_0 \frac{\delta \bar{H}}{\delta S_k} \right) + \tfrac{1}{2} \Gamma_0 \sum_p \frac{\delta^2 \bar{H}}{\delta S_p \delta S_{-p}} \Bigg\} \tag{2.9}$$

where $\tilde{Q}_{k,\lambda}$ and $\tilde{S}_k$ are auxiliary 'response' fields. With the Lagrangian, all correlation and response functions can be computed as path integrals weighted with density $\exp(L)$.

Next we apply the RNG method to the full Lagrangian, integrating out all $\tilde{S}$, S, $\tilde{Q}$, Q with momenta and frequencies in the region

$$\mathrm{e}^{-l} < k < 1 \qquad -\infty < \omega < \infty.$$

To first order in $\varepsilon = 4 - d$ the relevant recursion relations are

$$\frac{\mathrm{d}r}{\mathrm{d}l} = 2r + \frac{3u + v}{1 + r} \tag{2.10a}$$

$$\frac{\mathrm{d}u}{\mathrm{d}l} = u\left(\varepsilon - \frac{9u}{(1 + r)^2}\right) \tag{2.10b}$$

$$\frac{\mathrm{d}v}{\mathrm{d}l} = v\left(\varepsilon - \frac{(6u + v)}{(1 + r)^2}\right) \tag{2.10c}$$

$$\frac{\mathrm{d}v^{\mathrm{ph}}}{\mathrm{d}l} = v^{\mathrm{ph}}\left(\varepsilon - \frac{(6u + v^{\mathrm{ph}})}{(1 + r)^2}\right) \tag{2.10d}$$

$$\frac{\mathrm{d}c^2}{\mathrm{d}l} = \frac{-v^{\mathrm{ph}} c^2}{(1 + r)^2} \tag{2.10e}$$

where $c^2 = C_{11}/\bar{\rho}$ is the square of the longitudinal velocity (the transverse modes are not coupled to the OP in this model) and the parameters u, v, v^{ph} are defined by

$$v^{\mathrm{ph}} = \frac{2g^2}{C_{11}} K_4 \qquad v = v^{\mathrm{ph}} - wK_4 \qquad u = \tilde{u}K_4 - v^{\mathrm{ph}}$$

and $K_4 = 1/(8\pi^2)$.

For the discussion of the OP properties it is sufficient to study only the first three of these equations. They can be obtained if we integrate out the elastic variables at the beginning of our calculations, as was done by Bruno and Sak (1980). They found four 'OP' fixed points in the space of parameters (r, u, v). These include two rigid fixed points $(v^* = 0)$: Gaussian (G) (0, 0, 0) and Ising (I) $(-\varepsilon/6, \varepsilon/9, 0)$; as well as the corresponding renormalised fixed points $(v^* \neq 0)$: spherical (S) $(-\varepsilon/2, 0, \varepsilon)$ and renormalised Ising (rI) $(-\varepsilon/3, \varepsilon/9, \varepsilon/3)$. The projections of the RNG flows and fixed points, in the plane $r =$ const, are shown schematically in figure 1. The most stable renormalised Ising fixed point is inaccessible because the bare value of the parameter v is negative for a solid

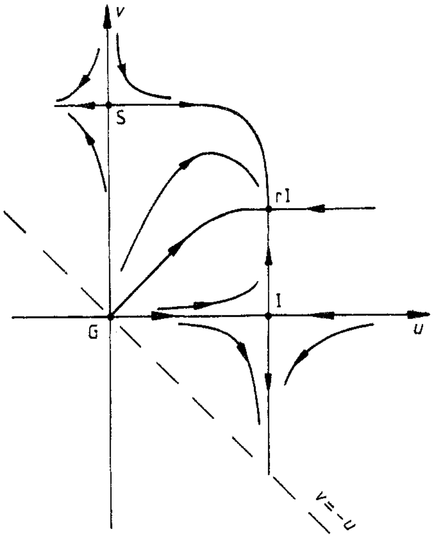

Figure 1. The projections of the RNG flows and fixed points onto the plane r = const.

with a positive value of the shear modulus. On the other hand the Ising fixed point is unstable against v perturbations with the scaling exponent $\lambda_v = \alpha/\nu$, where α and ν are Ising exponents. The observed run-away was interpreted as a signal of a first-order transition (Sak 1974), the physical nature of which is associated with the fact that the bulk modulus

$$B = C_{11} - \frac{2(d-1)}{d} C^0_{44}$$

becomes negative close to T_c, indicating an instability against certain macroscopic deformations. However, if v_0 is small, the Ising fixed point will determine the behaviour of the system over a wide range of experimentally controllable parameters (like reduced temperature). This pseudo-critical region may be quite large because of the smallness of λ_v.

The subspace of parameters (r, u, v) is insufficient to permit a description of the acoustic properties of the system. It turns out that in order to study these properties one has to take into account another relevant parameter, $v^{\rm ph}$, which fulfils the same RNG equation as v, but has a different initial value. Contrary to v_0, the bare value of $v^{\rm ph}$ is always positive. Moreover, the absolute value of the former may be much smaller than the latter if $C^0_{44} \ll C^0_{11}$. Thus, under repeated RNG transformation, $v^{\rm ph}$ will approach a fixed-point value $v^{\rm ph*}_{\rm I} = \alpha/\nu + O(\varepsilon^2)$. The fixed point $(r^*_{\rm I}, u^*_{\rm I}, v^*_{\rm I}, v^{\rm ph*}_{\rm I})$ in the space $(r, u, v, v^{\rm ph})$ will be denoted as $\rm I_2$, to be distinguished from $(r^*_{\rm I}, u^*_{\rm I}, v^*_{\rm I}, 0)$, denoted as $\rm I_1$. The description in terms of the two parameters v and $v^{\rm ph}$ is sensible only for the system at a constant pressure, where their bare values can be treated to a certain degree as independent. If a constant-volume constraint is imposed on the system, then these parameters become identical.

Now we shall concentrate on the evaluation of the dynamic response function of the longitudinal phonons $G(k, \omega)$, from which the sound velocity and attenuation can be extracted. We have applied the matching method introduced in statics by Nelson (1976). This method was first applied to dynamics by Siggia and Nelson (1977) and by Siggia (1977), who found the temperature dependence of some kinetic coefficients at zero momentum and frequency, and then it was generalised to finite momenta and frequencies by Folk *et al* (1977). To start with we need to solve the recursion relations to $O(\varepsilon)$:

$$\tau_l = r_l + \tfrac{1}{2}(3u_{\rm I}^* + v_l)(1 - r_l)\ln(1 + r_l) \tag{2.11a}$$

$$v_l = v_0\, e^{(\alpha/\nu)l}\,[1 - v_0/v_{\rm rI}^* + (v_0/v_{\rm rI}^*)\, e^{(\alpha/\nu)l}]^{-1} \tag{2.11b}$$

$$v_l^{\rm ph} = v_0^{\rm ph}\, e^{(\alpha/\nu)l}[1 - v_0^{\rm ph}/v_{\rm I}^{\rm ph*} + (v_0^{\rm ph}/v_{\rm I}^{\rm ph*})\, e^{(\alpha/\nu)l}]^{-1} \tag{2.11c}$$

$$c_l^2 = c_0^2[1 - v_0^{\rm ph}/v_{\rm I}^{\rm ph*} + (v_0^{\rm ph}/v_{\rm I}^{\rm ph*})\, e^{(\alpha/\nu)l}]^{-1} \tag{2.11d}$$

where the temperature scaling field is defined by

$$\tau_l = \tau\, e^{l/\nu}[1 - v_0/v_{\rm rI}^* + (v_0/v_{\rm rI}^*)\, e^{(\alpha/\nu)l}]^{-1}. \tag{2.12}$$

Here $\tau \equiv \tau_0 \sim (T - T_c)$ and α, ν are the rigid Ising exponents. In the above expression for simplicity we keep the parameter u at its fixed-point value $u_{\rm I}^*$.

With this solution at hand we now proceed to calculate the acoustic response function

$$G(k, \omega) = \langle \tilde{Q}_{k,\omega} Q_{-k,-\omega}\rangle_L. \tag{2.13}$$

The recursion relation method for any physical quantity involves making use of the scaling relation it satisfies. In this case we use the relation

$$G(k, \omega; \{\mu\}) = e^{2l}\, G(k_l, \omega_l; \{\mu_l\}) \tag{2.14}$$

where $k_l = k\, e^l$, $\omega_l = \omega\, e^{zl}$ and $\{\mu_l\}$ is a set of parameters of the Lagrangian at the lth stage of iteration with $\mu \equiv \mu_0$.

The effect of couplings in $L(l)$ on the response function $G(l)$ may be expressed in terms of a self-energy $\Sigma(l)$ given by

$$G^{-1}(l) = G_0^{-1}(l) + \Sigma(l) \tag{2.15}$$

where the dependence on k_l and ω_l has been dropped for simplicity and $G_0(l)$ is the response function in the harmonic approximation for the Lagrangian. $G_0^{-1}(l)$ may be expressed as

$$G_0^{-1}(l) = c_0^2 k_l^2 - {\rm i}\, e^{-zl}\, D^0 k_l^2 \omega_l - e^{-(2z-2)l}\, \omega_l^2 - 2\int_0^l {\rm d}l'\, g_{l'}^2 k_{l'}^2\, e^{-2l'}$$
$$\times [(1 + r_{l'} - {\rm i}\omega_{l'}/\Gamma_0)(1 + r_{l'})\bar{\rho}]^{-1} \tag{2.16}$$

where the first line represents the bare response function after a simple change of scale and the second one comes from the trajectory integral representing contributions generated by the iterations of the RNG transformations.

Since the Lagrangian is bilinear in Q and $\tilde{Q}$, it may be split into two independent parts

$$L[S, \tilde{S}, Q, \tilde{Q}] = L_{\rm OP}^{\rm eff}[S, \tilde{S}] + L[Q, \tilde{Q}]$$

using the transformation

$$Q_{k,\omega} \to Q_{k,\omega} - f_k G_0(k, \omega)(S^2)_{k,\omega} - 4f_{-k} D^0 k^2 \Gamma_0 |G_0(k, \omega)|^2 (\tilde{S}S)_{k,\omega}$$
$$\tilde{Q}_{k,\omega} \to \tilde{Q}_{k,\omega} - 2\Gamma_0 f_k G_0(-k, -\omega)(\tilde{S}S)_{k,\omega} \tag{2.17}$$

where $|f_k|^2 = g^2k^2/\bar{\rho}$ and the l dependence has been suppressed. The OP effective Lagrangian $L_{\rm OP}^{\rm eff}$ contains the strain mediated interactions, whose effect can be represented simply by a shift of the parameters

$$\tilde{u} \rightarrow u = \tilde{u} - v^{\mathrm{ph}} \qquad w \rightarrow v = w - v^{\mathrm{ph}}. \tag{2.18}$$

With the help of transformation (2.17) the self-energy $\Sigma(l)$ may be expressed entirely in terms of the OP 'energy' response function $D(k, \omega; l)$

$$\bar{\rho}\Sigma(k_l, \omega_l; l) = -2g_l^2 k_l^2 \frac{D(k_l, \omega_l; l)}{1 + v_l^{\mathrm{ph}} D(k_l, \omega_l; l)} \tag{2.19}$$

where

$$D(k, \omega; l) = \langle \Gamma_0 (\tilde{S}S)_{k,\omega} (S^2)_{-k,-\omega} \rangle_{L_{\mathrm{OP}(l)}^{\mathrm{eff}}}. \tag{2.20}$$

The denominator in (2.19) is necessary so as not to include the reducible part of the phonon propagator to Σ. It was neglected in the lowest order in the elastic coupling theories (Murata 1976, Iro and Schwabl 1983, Fossum 1985).

The scaling relation (2.14) as well as (2.16) and (2.19) lead to the result

$$G^{-1}(k, \omega) = c_l^2 k^2 - c_l^2 k^2 \frac{v_l^{\mathrm{ph}} D(k_l, \omega_l)}{1 + v_l^{\mathrm{ph}} D(k_l, \omega_l)} - k^2 \int_0^l \mathrm{d}l' \, v_{l'}^{\mathrm{ph}} c_{l'}^2 \times \{[(1 + r_{l'} - \mathrm{i}\omega_{l'}/\Gamma_0)(1 + r_{l'})]^{-1} - 1\} - \omega^2 \tag{2.21}$$

where the leading contribution from the trajectory integral has been incorporated into c_l^2.

Still we have to determine the energy response function D for some $l = l^*$, where perturbation theory could be used. In the ultrasonic experiments the wavelength is much longer than the correlation length so the average in (2.20) may be evaluated in the limit $k = 0$. Following Dengler *et al* (1985) we use the matching condition

$$(\omega_{l^*}/2\Gamma_0)^{4/z} + \chi_{l^*}^{-2} = 1 \tag{2.22}$$

with the static OP susceptibility obeying the scaling relation

$$\chi = \mathrm{e}^{2l} \chi_l \tag{2.23}$$

where χ may be calculated by the static version of this method (Bruno and Sak 1980) with the result

$$\chi = \tau^{-\gamma}(1 - b + b\tau^{-\alpha}) \tag{2.24}$$

where $b = v_0/v_{\mathrm{II}}^*$ and $\gamma = 2\nu + O(\varepsilon^2)$.

The matching condition (2.22) permits an explicit solution for l^*

$$\mathrm{e}^{l^*} = \tau^{-\nu} Q(y; \tau) \tag{2.25}$$

with

$$Q(y; \tau) = [(1 - b + b\tau^{-\alpha})^{-2} + (y/2)^{4/z}]^{-1/4}$$

and $y = \omega\tau^{-z\nu}/\Gamma_0$ as the reduced frequency.

The function D has been calculated perturbatively by Iro and Schwabl (1983) in a rigid system (function Σ_1 in that paper). The diagrams are shown in figure 2. For the internal lines, in principle, one would have to take an expression valid in the whole range of k, ω and τ (e.g. the result of Dengler *et al* (1985)). But this can be done only

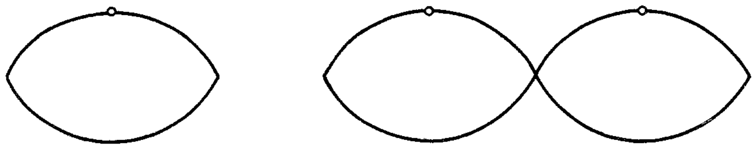

Figure 2. Diagrams contributing to the self-energy in the first order in ε. The curves and the circled curves represent the OP response and the correlation functions, respectively.

numerically. Iro and Schwabl (1983) have approximated the internal propagators by the ones for which self-energy is restricted to the Hartree bubble. The fixing condition for the external parameters ω and τ reduces the region in the integral corresponding to the diagram, where the difference really matters, leading to the result being in very good agreement with numerical results (Iro 1984). In the present paper we have used another approximation. Because the low-frequency and small-momentum behaviour of the OP propagators contribute most significantly to the integral, we have used the conventional approximation for the OP response function

$$G_{\bar{s}s}(p, \nu; l) = \chi_l^{-1} + p^2 - \mathrm{i}\nu/\Gamma_0 \tag{2.26}$$

which together with the matching condition (2.22) is sufficient to obtain the small-y behaviour of $d(y) \equiv D(yQ^z(y)\Gamma_0, 0)$ correct to $O(\varepsilon)$. However, in both approximations the large-y behaviour of $d(y)$ can be determined only to leading order in y^{-1}. Fortunately, the high-frequency corrections to the leading behaviour are small, so their detailed form is not very important in this problem.

Next, we substitute (2.25) into (2.21). From the renormalised phonon frequency we obtain the expression for the sound velocity

$$c^2(\omega, \tau) = \frac{c_0^2}{R}\left(1 - \frac{N}{R}\operatorname{Re}(\tilde{d}^0)\right) + O(\varepsilon^2) \tag{2.27a}$$

and the corresponding phonon width $\Gamma_{\mathrm{ph}}(\omega, \tau)$ determines the attenuation coefficient

$$\alpha(\omega, \tau) = \frac{\Gamma_{\mathrm{ph}}(\omega, \tau)}{2c(\omega, \tau)} = J\omega^2\tau^{-(z\nu+\alpha)}Q^{\alpha/\nu}y^{-1}R^{-1/2}$$
$$\times\left(\operatorname{Im}(\tilde{d}) - \frac{N}{2R}\operatorname{Im}(\tilde{d}^0)\operatorname{Re}(\tilde{d}^0)\right) + O(\varepsilon^2) \tag{2.27b}$$

where we have ignored the regular contributions, J is a non-universal constant and the abbreviations are $N = v_{\mathrm{I}}^{\mathrm{ph}*}a\tau^{-\alpha}Q^{\alpha/\nu}$, $R = 1 - a + (N/v_{\mathrm{I}}^{\mathrm{ph}*})$ and $a = v_0^{\mathrm{ph}}/v_{\mathrm{I}}^{\mathrm{ph}*}$. The function $\tilde{d}$, which is a sum of function d and the remainder of the trajectory integral in (2.21), may be written as

$$\tilde{d}(\hat{y}) = \tilde{d}^0 - \frac{\mathrm{i}\varepsilon}{4y}\left[4\ln^2(Q) + \left(1 - \mathrm{i}\frac{\hat{y}}{2}\right)A^2 + \mathrm{i}\hat{y}\frac{\pi^2}{6}\right] - 3u_{\mathrm{I}}^*(\tilde{d}^0)^2 + O(\varepsilon^2) \tag{2.28}$$

with

$$\tilde{d}^0 = \frac{\mathrm{i}}{\hat{y}}\left[2\frac{\ln(\hat{Q})}{\hat{y}} + \mathrm{i}\left(1 - \mathrm{i}\frac{\hat{y}}{2}\right)A\right] \qquad A = \tan^{-1}\left(\frac{\hat{y}}{2}\right).$$

In the above procedure we encounter the same difficulties as in the static case (Bruno and Sak 1980), associated with crossing a mean-field instability line $u = -v$ under repeated iterations of the RNG, before the condition (2.22) is satisfied. An inequality $\tau^{\nu} Q^{-1}(y) \geq K$ where

$$K = (1-h)(1-b)\left|\frac{b}{(1-b)h}\right|^{\nu/\alpha} \tag{2.29}$$

with $h = u_{\mathrm{I}}^{*}/(u_{\mathrm{I}}^{*} + v_{\mathrm{rI}}^{*})$, must be fulfilled in order that the results (2.27) remain valid. The same reasoning concerning the static susceptibility in (2.24) leads to the even more restrictive condition $\tau \geq \tau_{\min}^{+}$, where

$$\tau_{\min}^{+} = K^{1/\nu} \tag{2.30}$$

indicates the onset of the first-order transition. For $\tau \leq \tau_{\min}^{+}$ other techniques have to be applied to calculate physical properties of the system (Nicoll 1981). In this paper we restrict our considerations to the region $\tau \geq \tau_{\min}^{+}$.

On the other hand the present description is expected to be valid in the range of parameters where the corrections to scaling due to the departure of the coupling u from the fixed-point value u^{*} (our analysis can easily be generalised to include these corrections) as well as higher transients and analytic corrections are still negligible. The first non-asymptotic analysis of critical sound propagation, valid in the entire range of applicability of the Ginzburg–Landau Hamiltonian, was performed by Pankert and Dohm (1986) for ^{4}He ($n = 2$).

3. Asymptotic behaviour

In this section we shall discuss the general expression (2.27). It shows two distinct asymptotic regimes depending on the relative size of τ, ω and the parameter a. In the limit $a \to 0$, or more generally, if the inequality

$$\tau^{\nu} Q^{-1}(y) \gg K_a \tag{3.1}$$

is fulfilled, where K_a is roughly equal to $a^{\nu/\alpha}$, we recover the results of Iro and Schwabl (1983)

$$\alpha(\omega, \tau) \sim \omega^{2} \tau^{-\rho_{\mathrm{A}}} g_{\mathrm{A}}(y) \tag{3.2a}$$

$$c^{2}(\omega, \tau) - c^{2}(0, \tau) \sim \tau^{-\alpha} f_{\mathrm{A}}(y) \tag{3.2b}$$

where $\rho_{\mathrm{A}} = z\nu + \alpha$, $g_{\mathrm{A}}(y) = y^{-1} Q^{\alpha/\nu} \operatorname{Im}(\tilde{d}) + \mathrm{O}(\varepsilon^{2})$ and $f_{\mathrm{A}}(y) = \tilde{f}_{\mathrm{A}}(y) - \tilde{f}_{\mathrm{A}}(0)$ with $\tilde{f}_{\mathrm{A}}(y) = -Q^{\alpha/\nu}[1 + v_{\mathrm{I}}^{\mathrm{ph}*} \operatorname{Re}(\tilde{d}^{0})] + \mathrm{O}(\varepsilon^{2})$. The scaling functions g and f have a subscript A in this 'weak-coupling' regime.

There is yet another pseudo-critical regime for $a \to 1$ or for $\tau^{\nu} Q^{-1}(y) \ll K_a$ and with $|b| \ll 1$. For this 'strong-coupling' regime we have obtained different results

$$\alpha(\omega, \tau) \sim \omega^{2} \tau^{-\rho_{\mathrm{B}}} g_{\mathrm{B}}(y) \tag{3.3a}$$

$$c^{2}(\omega, \tau) - c^{2}(0, \tau) \sim \tau^{\alpha} f_{\mathrm{B}}(y) \tag{3.3b}$$

where $\rho_{\mathrm{B}} = z\nu + (\alpha/2)$

$$g_{\mathrm{B}}(y) = y^{-1} Q^{\alpha/2\nu}[\operatorname{Im}(\tilde{d}) - (v_{\mathrm{I}}^{\mathrm{ph}*}/2) \operatorname{Re}(\tilde{d}^{0}) \operatorname{Im}(\tilde{d}^{0})] + \mathrm{O}(\varepsilon^{2})$$

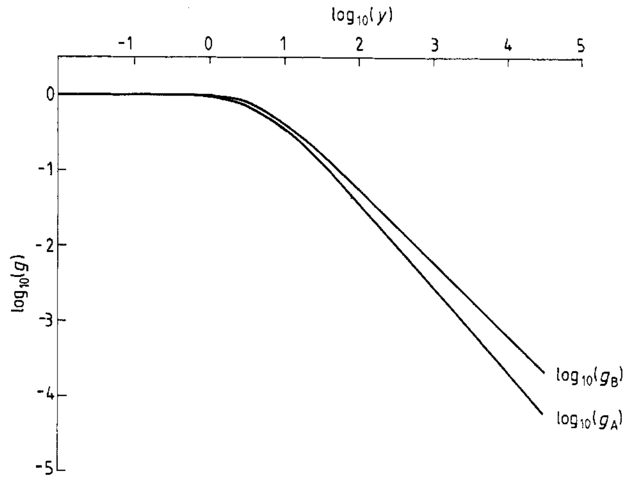

Figure 3. Scaling functions for sound attenuation for $\varepsilon = 1$ with normalisation $g_A(0) = g_B(0) = 1$.

and $f_B(y) = \tilde{f}_B(y) - \tilde{f}_B(0)$ with $\tilde{f}_B(y) = Q^{-\alpha/\nu}[1 - v_I^{ph*}\ \mathrm{Re}(\tilde{d}^0)] + O(\varepsilon^2)$. The scaling functions g and f for both regimes are compared in figures 3 and 4. In figure 3 the flat parts of the curves correspond to the hydrodynamic region ($y \ll 1$) where the leading behaviour

$$\alpha(\omega, \tau) \sim \omega^2 \tau^{-\rho} \tag{3.4}$$

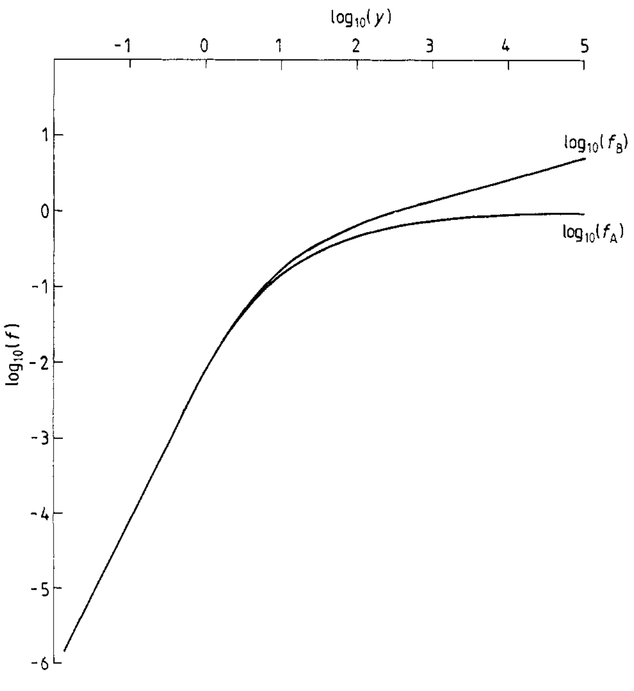

Figure 4. Scaling functions for sound dispersion. The normalisation $f_A(0) = f_B(0)$ and $f_A(\infty) = 1$ has been used.

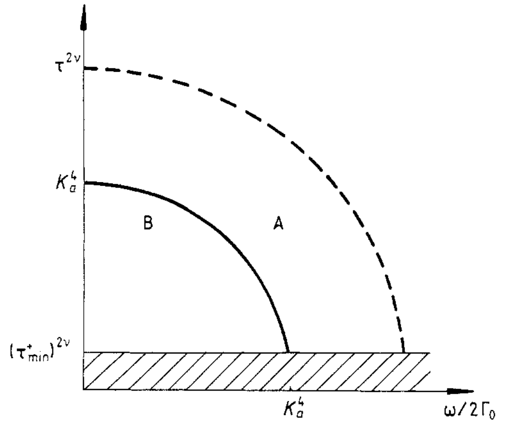

Figure 5. Schematic division of $(\tau^{2\nu}, \omega/2\Gamma_0)$-plane into weak- and strong-coupling regimes. The hatched area $\tau \leqslant \tau^+_{\min}$ lies outside the region of validity of the method used in this paper.

is found with the corresponding value of the exponent ρ for given regime. For $y \to \infty$ (critical region) we obtain

$$\alpha(\omega, \tau) \sim \omega^{2-(\rho/\nu z)}. \tag{3.5}$$

In figure 4 the common part of both curves corresponds to the hydrodynamic behaviour $f(y) \sim y^2$. For $y \to \infty$, $f_A(y)$ saturates as $[1 - (2/y)^{\alpha/z\nu}]$ while $f_B(y)$ diverges as $[(2/y)^{-\alpha/z\nu} - 1]$.

The pseudo-critical behaviour can be observed if $|b| \ll 1$. In this case $\tau^+_{\min}$ is small. In order to fulfil this condition as well as $a \simeq 1$, simultaneously, $C^0_{44} \ll C^0_{11}$ must hold. Thus we expect that the expressions (3.2) are especially suitable for the systems with a strong OP–phonon coupling and with a small ratio of shear and longitudinal moduli.

The 'strong-coupling' regime results were previously obtained (Pawlak 1984) by the use of Wilson–Feynman technique (Wilson 1972). In that method one artificially extends the pseudo-critical 'strong-coupling' region by putting $u_0 = u^*_{\rm I}$, $v_0 = 0$ and $v^{\rm ph}_0 = v^{\rm ph*}_{\rm I}$. Then the singular logarithmic terms may be exponentiated and the correct leading behaviour is obtained.

The inequality (3.1) may be interpreted graphically as is shown in figure 5 where the full curve symbolises a cross-over region from weak- to strong-coupling regime. In the hatched area $\tau \leqslant \tau^+_{\min}$, near the first-order transition, the formulae obtained in this paper are not valid. The broken curve separates the weak-coupling regime from a non-critical region.

4. Non-asymptotic behaviour

In more realistic systems the observed behaviour may be intermediate between the weak- and strong-coupling regimes. Moreover, as long as the first-order transition region is approached the macroscopic instability effects become apparent. In the following, we shall study such non-asymptotic behaviour on the basis of (2.27). It should be stressed that the term 'non-asymptotic' is used in this paper in a slightly different sense from its usual meaning of 'not asymptotically close to criticality'. Here by 'asymptotic' we understand the behaviour fully determined by one of the fixed points (I_1 or I_2) and expected to be characteristic within a certain range of physical parameters. The main point is that our asymptotic behaviours are not actually asymptotic because if τ decreases below $\tau^+_{\min}$ the system evolves eventually towards a first-order transition.

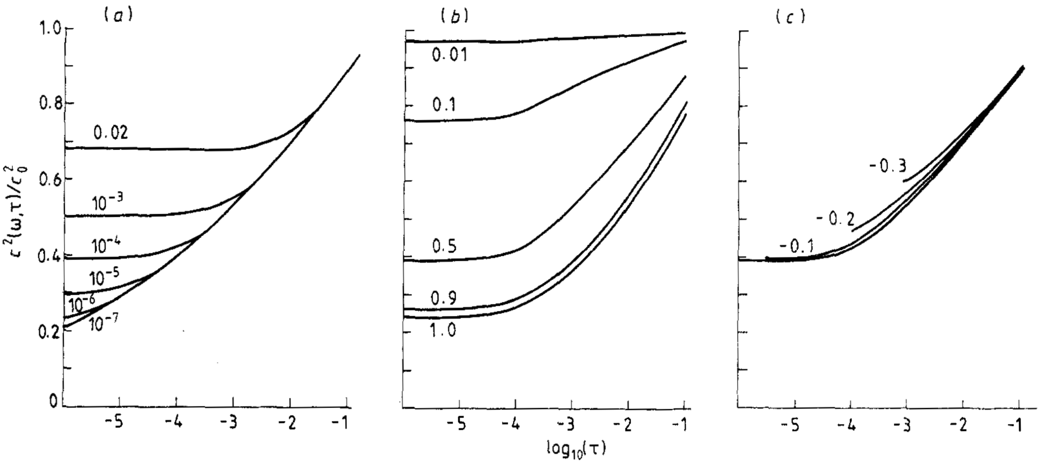

Figure 6. The temperature dependence of the square of sound velocity: (*a*) for several values of ω/Γ_0 with $a = 0.5$ and $b = 0$; (*b*) for several vaues of the parameter a with $\omega/\Gamma_0 = 10^{-4}$ and $b = 0$; and (*c*) for several values of b with $\omega/\Gamma_0 = 10^{-4}$ and $a = 0.5$. The figure is obtained from (2.27*a*) with $\varepsilon = 1$.

The expressions (2.27) are illustrated in figures 6 and 7, where we have plotted $\alpha(\omega, \tau)$ and $c^2(\omega, \tau)$ as functions of $\log_{10}(\tau)$ for different values of the parameters a, b and ω/Γ_0, with $\varepsilon = 1$. As follows from these figures for small values of a, the sound velocity changes only slightly in the whole temperature interval considered. Its level of saturation decreases as the frequency ω/Γ_0 goes to zero, but for finite ω/Γ_0 the sound velocity remains finite even very close to the transition temperature. The slope of the sound attenuation coefficient in the hydrodynamic region depends on a and b (figure 7).

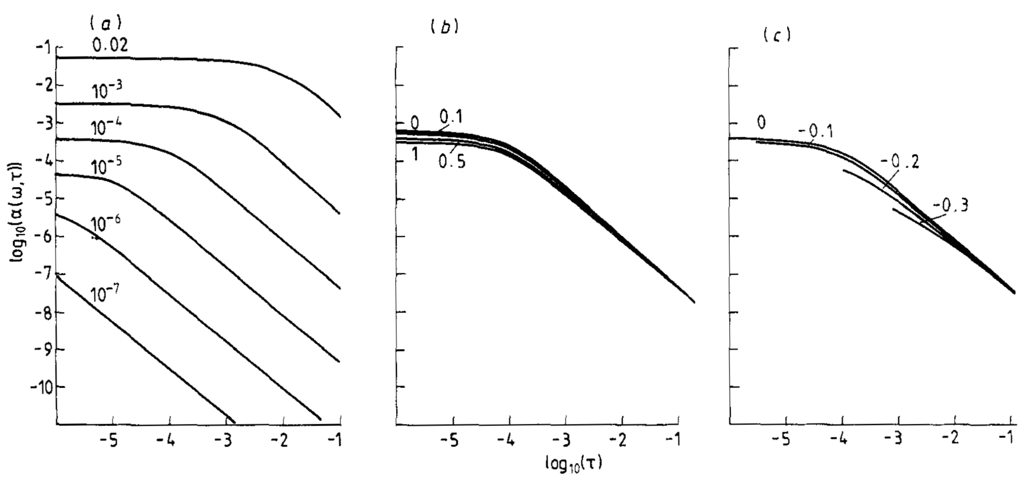

Figure 7. Double logarithmic plot of the sound attenuation coefficient from (2.27*b*) with $J\Gamma_0^2 = 1$ and $\varepsilon = 1$: (*a*) for various values of ω/Γ_0 with $a = 0.5$ and $b = 0$; (*b*) for various values of a with $\omega/\Gamma_0 = 10^{-4}$ and $b = 0$; and (*c*) for various values of b with $\omega/\Gamma_0 = 10^{-4}$ and $a = 0.5$.

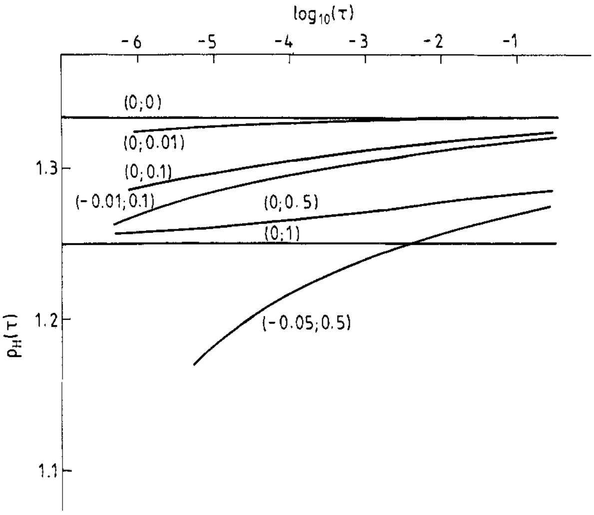

Figure 8. Effective hydrodynamic sound attenuation exponent plotted against $\log_{10}(\tau)$ for a variety of different values of $(b; a)$ from (4.2). The curves with (0; 0) and (0; 1) correspond to the asymptotic values ρ_A and ρ_B, respectively.

One can define an effective hydrodynamic sound attenuation exponent

$$\rho_H(\tau) = -\frac{\partial\{\log[\alpha(\omega, \tau)_{y\to 0}]\}}{\partial \log(\tau)} \tag{4.1}$$

which characterises the initial increase in the attenuation coefficient. From (2.27*b*) we have

$$\rho_H(\tau) = z\nu + \alpha - \frac{\alpha}{2}\frac{a\tau^{-\alpha}}{1 - a + a\tau^{-\alpha}} + \alpha\frac{b\tau^{-\alpha}}{1 - b + b\tau^{-\alpha}} + O(\varepsilon^2). \tag{4.2a}$$

$\rho_H(\tau)$ is shown in figure 8 as a function of $\log_{10}(\tau)$ for various values of $(b; a)$. The lines with (0; 0) and (0; 1) correspond to the weak- and strong-coupling regimes, respectively.

It is interesting to test (4.2) in the case of periodic boundary conditions (V = const). Then $b = a$ and at the renormalised Ising fixed point we obtain

$$\rho_H = z\nu + 3\alpha/2 + O(\varepsilon^2) \tag{4.2b}$$

since $a \to 1$. On the other hand if we put Fisher-renormalised exponents α and ν into the equation $\rho_H = z\nu + \alpha$, expected to be valid in the 'weak-coupling' regime, i.e. $\alpha \to -\alpha/(1-\alpha) \simeq -\alpha$ and $\nu \to \nu/(1-\alpha) \simeq \nu + \alpha/2$, then we obtain $\rho_H = z\nu + O(\varepsilon^2)$. The difference between these values is due to the singular behaviour of longitudinal sound velocity ($c^2 \sim C_{11} \sim \tau^{\alpha}$), which tends to zero for a compressible Ising system with a constant volume (Bergman and Halperin 1976). It is easy to check that the very third power of sound velocity that appears in the denominator of the expression for sound attenuation (e.g. equation (6) in the paper of Iro and Schwabl (1983)) is responsible for the additional term $3\alpha/2$ in equation (4.2*b*).

The curves in figures 6, 7 and 8 have physical interpretation only for $\tau \geqslant \tau^+_{\min}$. The value of $\tau^+_{\min}$ is very sensitive to the choice of parameter h in (2.29). When u_I^* and v_{rI}^* are evaluated to $O(\varepsilon)$, h is 0.25, but if we evaluate u_I^* and v_{rI}^* to the second order in ε (de Moura *et al* 1976) then $h = 0.65$ and the values of $\tau^+_{\min}$ are much smaller than those calculated for $h = 0.25$, as is shown in table 1. Equations (2.29) and (2.30) imply that the relatively large value of the exponent α in the $O(\varepsilon)$ approximation also overestimates the size of the region where the first-order nature of the transition becomes apparent.

Table 1. The temperature parameter $\tau^{+}_{\min}$ estimated from (2.30) for a few values of b. The specific heat exponent $\alpha = \varepsilon/6$ and $\varepsilon = 1$.

	$\tau^{+}_{\min}$	
b	$h = 0.25$	$h = 0.65$
-0.05	4×10^{-5}	6×10^{-8}
-0.10	2×10^{-3}	3×10^{-6}
-0.15	2×10^{-2}	3×10^{-5}
-0.20	8×10^{-2}	1×10^{-4}
-0.30	0.6	9×10^{-4}

The scaling representation of our results may be obtained by introducing non-linear scaling fields appropriate to this problem. They are functions of the physical fields τ, v_0 and v^{ph}_0 and are defined by

$$g_\tau(l) = g_\tau \exp(\lambda_\tau l) \qquad g^{\mathrm{ph}}_v(l) = g^{\mathrm{ph}}_v \exp(\lambda^{\mathrm{ph}}_v l) \qquad g_v(l) = g_v \exp(\lambda_v l) \tag{4.3}$$

where $\lambda_\tau = 1/\nu$ and $\lambda_v = -\lambda^{\mathrm{ph}}_v = \alpha/\nu$ are the corresponding exponents associated with linear scaling fields at the fixed point I_2. The non-linear fields determined by (4.3) are

$$g_\tau = \frac{\tau}{1-b} \qquad g^{\mathrm{ph}}_v = \frac{1-a}{a} \qquad g_v = \frac{b}{1-b} \tag{4.4}$$

in terms of which the solutions of the recursion relations become

$$\tau(l) = \frac{g_\tau(l)}{1+g_v(l)} \qquad v(l) = \frac{g_v(l)v^{*}_{\mathrm{rI}}}{1+g_v(l)} \qquad v^{\mathrm{ph}}(l) = \frac{g^{\mathrm{ph}}_v(l)v^{*}_{\mathrm{rI}}}{1+g^{\mathrm{ph}}_v(l)} \tag{4.5}$$

and the solution of equation (2.22) can be written as

$$e^{l^*} = g_\tau^{-\nu}[(1+n)^{-4\nu} + (y/2)^2]^{-1/4} \tag{4.6}$$

where now $y = (\omega/\Gamma_0)g_\tau^{-z\nu}$. The variable $n = g_v g_\tau^{-\phi_v}$ with $\phi_v = \lambda_v/\lambda_\tau = \alpha$ measures the distance from the first-order transition. Substituting these results into (2.27) we obtain various acoustic quantities in the scaling forms

$$\alpha(\omega,\tau) \sim g_\tau^{-\rho_B}\omega^2 F_\alpha(y;q,n) \tag{4.7a}$$

$$c^2(\omega,\tau) - c^2(0,\tau) \sim g_\tau^{\alpha} F_{\Delta 1}(y;q,n) \tag{4.7b}$$

$$c^{-2}(0,\tau) - c^{-2}(\omega,\tau) \sim g_\tau^{-\alpha} F_{\Delta 2}(y;n) \tag{4.7c}$$

where the variable $q = g^{\mathrm{ph}}_v g^{\alpha}_\tau$ describes the cross-over from I_1 ($q = \infty$) to I_2 ($q = 0$). The last two expressions are useful for a description of sound dispersion. Contrary to $F_{\Delta 1}$ the scaling function $F_{\Delta 2}$ does not depend on the cross-over variable in this order of approximation.

Next the n dependence of the scaling functions may be eliminated by a renormalisation of the variables

$$y \to \tilde{y} = y(1+n)^{z\nu} = (\omega/\Gamma_0)\tau^{-z\nu_{\text{eff}}(\tau)} \tag{4.8a}$$

$$q \to \tilde{q} = q(1+n)^{-\alpha} \tag{4.8b}$$

$$g_\tau^\nu \to g_\tau^\nu(1+n)^{-\nu} = \tau^{\nu_{\text{eff}}(\tau)} \tag{4.8c}$$

where we have introduced an effective exponent $\nu_{\text{eff}}(\tau)$ (Riedel and Wegner 1974), which can be defined here by

$$\nu_{\text{eff}}(\tau) = -\frac{1}{z}\frac{\partial \log(\tilde{y})}{\partial \log(\tau)}. \tag{4.9}$$

It is obtained to order ε with the result

$$\nu_{\text{eff}}(\tau) = \nu + \frac{\alpha}{2}\frac{b\tau^{-\alpha}}{1-b+b^{-\alpha}}. \tag{4.10}$$

Finally, instead of (4.7) we obtain

$$\alpha(\omega,\tau) \sim \tau^{-\rho_{\text{H}}(\tau)}\tilde{F}_\alpha(\tilde{y};\tilde{q}) \tag{4.11a}$$

$$c^2(\omega,\tau) - c^2(0,\tau) \sim \tau^{\alpha}\tilde{F}_{\Delta1}(\tilde{y};\tilde{q}) \tag{4.11b}$$

$$c^{-2}(0,\tau) - c^{-2}(\omega,\tau) \sim \tau^{-\alpha}\tilde{F}_{\Delta2}(\tilde{y}) \tag{4.11c}$$

where the scaling functions with a tilde, $\tilde{F}_\alpha(\tilde{y};\tilde{q}) = F_\alpha(\tilde{y};\tilde{q},0)\ldots$, are plotted in figures 9 and 10 for various values of the cross-over parameter $\tilde{q}$. The scaling functions

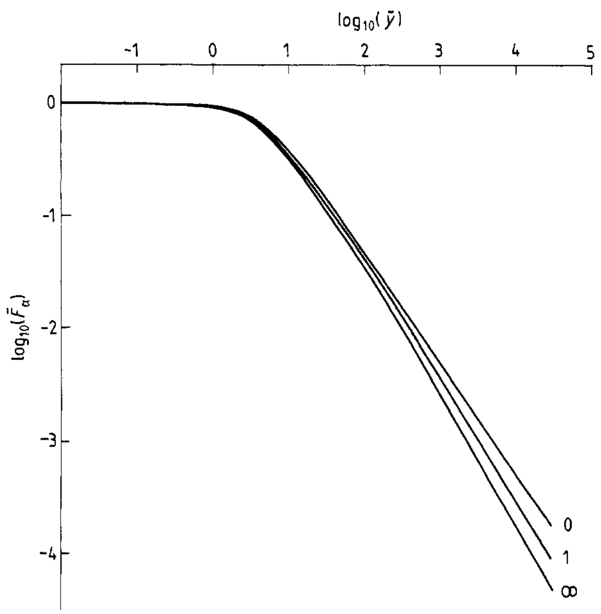

Figure 9. The function $\log_{10}(\tilde{F}_\alpha)$ plotted against $\log_{10}(\tilde{y})$ for different values of $\tilde{q}$, where we have normalised $\tilde{F}_\alpha$ to be unity at $\tilde{y} = 0$. The limiting values 0 and ∞ of $\tilde{q}$ correspond to the scaling functions g_{B} and g_{A}, respectively.

$\tilde{F}_\alpha$ and $\tilde{F}_{\Delta 1}$ coincide with those of the strong-coupling regime g_B and f_B at $\tilde{q} = 0$, while in the limit $\tilde{q} \to \infty$ they are equal to the functions g_A and f_A, respectively. In the weak-coupling regime we have $\tilde{F}_{\Delta 1} \sim \tilde{F}_{\Delta 2}$ as should be expected.

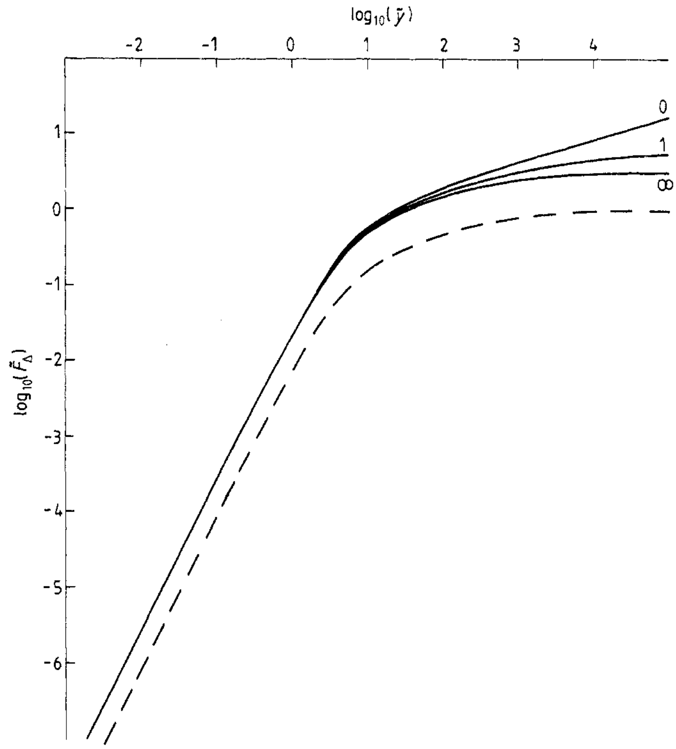

Figure 10. Double logarithmic plots of $\tilde{F}_{\Delta 2}$ (broken curve) and $\tilde{F}_{\Delta 1}$ (full curves) for different values of $\tilde{q}$. We have taken the normalisation $\tilde{F}_{\Delta 1}(0; \tilde{q}) = 10^{1/2}\tilde{F}_{\Delta 2}(0)$ and $\tilde{F}_{\Delta 2}(\infty) = 1$.

5. Tricritical behaviour

In consequence of the interaction between OP and the strain, the transition temperature is pressure-dependent and the effective coupling constant is shifted to $u_0 = \tilde{u}_0 - v_0^{\text{ph}}$ and thus may become negative at a certain temperature (u gains a temperature dependence if we maintain the previously neglected factor $k_B T$ in the explicit form). That is the reason why pseudo-critical behaviour may be expected in systems with strong OP–strain coupling (Bergman and Halperin 1976, Bruce 1980). Such a pseudo-tricritical point would separate a line of pseudo-critical points with very small jumps of entropy from a line of markedly first-order transition points with considerable entropy jumps. An analysis, similar to that shown explicitly in § 2 (Pawlak 1987), leads straightforwardly to the conclusion that the acoustic pseudo-tricritical behaviour is controlled by one of two Gaussian fixed points in (r, u, v, v^{ph}) space: G_1 $(0, 0, 0, 0)$ and G_2 $(0, 0, 0, \varepsilon)$.

The corresponding predictions for the sound attenuation and dispersion in the weak-coupling pseudo-tricritical regime, controlled by G_1, are

$$\alpha(\omega, \tau) \sim \tau^{-\rho^{t}_{A}} g^{t}_{A}(y) \tag{5.1a}$$

$$c^2(\omega, \tau) - c^2(0, \tau) \sim \tau^{-\alpha_t} f^{t}_{A}(y) \tag{5.1b}$$

while the analogous results in the strong-coupling regime (controlled by G_2) are

$$\alpha(\omega, \tau) \sim \tau^{-\rho^{t}_{B}} g^{t}_{B}(y) \tag{5.2a}$$

$$c^2(\omega, \tau) - c^2(0, \tau) \sim \tau^{\alpha_t} f^{t}_{B}(y) \tag{5.2b}$$

where $\rho^{t}_{A} = z_t \nu_t + \alpha_t$ and $\rho^{t}_{B} = z_t \nu_t + (\alpha_t/2)$ and α_t, ν_t, z_t are the usual (Gaussian) tricritical exponents. The scaling functions can be easily obtained from (2.27) and (2.28) by replacing u^*_I, v^{ph*}_I and the Ising exponents by their Gaussian values.

It is difficult to distinguish experimentally the acoustic critical behaviour in weak- and strong-coupling regimes, because the Ising exponent α is rather small. Fortunately, at the tricritical point the situation is more favourable because the tricritical exponent $\alpha_t = 0.5$ in three dimensions. Thus, one may hope that the different types of tricritical behaviour should be experimentally distinguishable. An example of a system displaying multicritical behaviour with the OP presumably strongly coupled to the acoustic phonons is some of the ammonium halides. The application of our theory to this case will be published in a separate paper.

6. Summary and conclusions

The aim of this paper was to investigate consistently the influence of elastic couplings on the critical and tricritical behaviour of the sound attenuation and dispersion. Our analysis shows that except for the conventional behaviour of these quantities described by (3.2) and (5.1) there is a new 'asymptotic' regime if $C^0_{44}/C^0_{11} \to 0$, described by (3.3) and (5.2). In general, a cross-over from the former to the latter regime is expected on which the macroscopic instability signals are superimposed. However, the temperatures in the vicinity of the first-order transition are outside the region of validity of our theory. Both effects, one due to the cross-over as well as that due to macroscopic instability, decrease the effective exponent controlling the growth of attenuation in the hydrodynamic region. The effective correlation length exponent may also be introduced in order to incorporate the latter effects. Then, the relatively simpler scaling forms of the acoustic quantities are obtained. The predicted new features would be observed in the systems in which appreciable velocity changes occur close to the transition point. The large specific heat exponent is also favourable for them. The most promising systems to verify our predictions would be those showing multicritical points. The measured sound attenuation tricritical exponents in ammonium halides are consistent with our findings.

The main emphasis of this paper was put on Ising-like systems above the transition temperature. However, the analysis developed here is also applicable to the ordered phase as well as to other systems including anisotropic coupling terms. In the latter case a general sound mode may be decomposed into symmetrised components with different sound attenuation and dispersion exponents (Fossum 1985). For example in the case of cubic systems (e.g. perovskites) the role of the specific heat exponent will be taken over by one of the $\alpha_i = \alpha + 2(\phi_i - 1)$, where $\phi_0 = 1$ and ϕ_1, ϕ_2 are the cross-over exponents associated with variables S_1S_2 and $S^2_1 - S^2_2$, respectively. Generally speaking the sound attenuation exponents in the 'strong-coupling' theories are expected to be smaller than the 'weak-coupling' ones.

Acknowledgments

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